Modified Split-Step Fourier Method for the Numerical Simulation of Soliton Amplification in Erbium-Doped Fibers with Forward Propagating Noise

Jorge R. Costa, Carlos R. Paiva, Afonso M. Barbosa

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Modified Split-Step Fourier Method for the Numerical Simulation of Soliton Amplification in Erbium-Doped Fibers with Forward-Propagating Noise

Jorge R. Costa, Member, IEEE, Carlos R. Paiva, Member, IEEE, and Afonso M. Barbosa, Senior Member, IEEE

Abstract—Using a modified version of the split-step Fourier method, we analyze the effect of noise on soliton propagation inside erbium-doped fiber amplifiers. In fact, noise from forward-propagating amplified spontaneous emission, associated with a Markov immigration process, is included in the analysis of soliton amplification. Moreover, this algorithm accounts for the real spectral gain profile of the fiber amplifier. The frequency jitter, induced during soliton amplification, is compared with the Gordon–Haus effect where optical amplifiers are considered as noisy point-like devices.

Index Terms—Amplified spontaneous emission, erbium-doped fiber amplifiers, fiber solitons, Gordon–Haus effect, split-step Fourier method.

I. INTRODUCTION

THE RECENT widespread research on optical amplifiers was mainly motivated by their transparency in optical networks: they provide format-independent gain, thus replacing the more expensive and limited electronic regenerators. In fact, they are responsible for the most relevant advances in lightwave systems for long-haul links. For fiber-optic communication systems, they can be used as power amplifiers to boost transmitter power, as in-line amplifiers to increase system reach, or as preamplifiers to enhance receiver sensitivity [1], [2].

Optical amplifiers can solve the fiber loss problem. However, the dispersive effects accumulate over long amplifier chains. Although several dispersion-compensating techniques have been developed for nonreturn-to-zero (NRZ) systems, optical solitons are probably the best solution to overcome the dispersion-limited performance of lightwave systems [3]–[5].

Pulse propagation in erbium-doped fiber amplifiers (EDFAs) requires a model that includes not only saturation but also the wavelength dependent gain. Soliton propagation, in particular, has been analyzed using a parabolic-gain profile as in the complex Ginzburg–Landau equation [6], [7] or a Lorentzian profile as in the Maxwell–Bloch equations [7], [8]. However, to study the amplification of wavelength-division-multiplexed (WDM) solitons inside an EDFA, these two profiles cannot be considered as reasonable models. To describe multichannel soliton amplification, the actual gain profile of the EDFA must be taken into account [9], [10].

An accurate analysis of solitons inside an EDFA should consider the continuously generated noise due to amplified spontaneous emission (ASE). We present, in this article, a generalization of the numerical method developed in [10], where only noise-free amplifiers were studied. We consider herein not only the effect of ASE on the amplifier’s gain, but also the effect of the forward-propagating noise on soliton amplification. Only when ASE noise is included, the random walk of amplified solitons can be predicted. Any realistic description of soliton transmission in fiber-optic communication systems with in-line optical amplification cannot disregard the timing jitter due to the Gordon–Haus effect [11], [12].

To analyze soliton propagation along nonlinear optical fibers, the most commonly used numerical algorithm is the split-step Fourier Method (SSFM) [7]. This method is based on nonlinear evolution equations. The simplest model corresponds to the nonlinear Schrödinger equation, which is only applicable when losses and optical amplification are disregarded. The SSFM, which may be considered as an alternative method to the beam propagation method (BPM), was not originally intended—see, e.g., [7]—to study the effect of ASE noise on soliton propagation in EDFAs. Nevertheless, these methods (such as BPM and SSFM) may be modified to solve stochastic partial differential equations [13].

There are, in the literature, several studies of soliton amplification as well as of noise in EDFAs. In [14], the authors studied pulse propagation in optical links with multiple EDFAs. Although they considered forward- and backward-propagating noise to obtain the amplifier characteristics, only the propagation of forward noise is included. Moreover, the amplifiers themselves were considered as point-like devices, i.e., at each amplifier its gain was applied to the input pulse and the ASE generated noise was added.

In this paper, the propagation inside the amplifier is considered and we include noise in each longitudinal step of the SSFM by using an iterative procedure to account for the forward ASE, associated with a Markov immigration process, generated in that step [13]. Forward and backward ASE are both taken into ac-
count to determine the spectral gain profile of the fiber amplifier. The main drawback of the present modified version of the SSFM is that in each longitudinal step, only the forward ASE noise can be included. However, as noted in [15], the use of an isolator at the middle of the amplifier can reduce the overall noise power, eliminate the accumulated backward-propagating noise and increase the available pump power at the beginning of the amplifier.

Due to ASE noise, there is a frequency jitter in solitons propagating inside the fiber amplifier. Since in our model neither the gain profile nor the noise power spectral density are flat, there is an average shift of the pulse’s central frequency. After discarding this frequency shift, we then compare the frequency jitter with the one predicted by the Gordon–Haus effect. To make that comparison, each iterative step in our numerical algorithm can be considered as an elementary amplifier where the Gordon–Haus jitter is generated.

WDM soliton propagation in EDFAs can be easily handled with our method, as shown by a simulated example with $4 \times 10^{10}$ Gbits/s WDM channels.

The remaining of the article is organized as follows. In Section II, we present the used model for the gain profile of an EDFA. An optimization procedure, to determine some relevant parameters of the fiber amplifier, is developed in Section III. The numerical algorithm for soliton propagation in the amplifier is explained in Section IV. The corresponding numerical results are presented in Section V. In Section VI, a soliton system with four WDM channels is simulated. Finally, in the last section, the conclusions are briefly outlined.

II. CHARACTERIZATION OF THE FIBER AMPLIFIER

To analyze soliton amplification, an accurate characterization of the fiber amplifier is needed. In this section, we describe the method that we used to obtain the gain coefficient $g(z, \omega)$ as well as the noise power spectral density $S_\eta(z, \omega)$.

There are two basic methods to characterize the amplifier: 1) using photon flux conservation [16] and 2) using a system of differential equations [2].

In the first method, the amplifier is divided into small sections. In each section the ion erbium density in the higher energy level is constant and the input/output photon relations are easily obtained. The entire amplifier is then described by a system of equations from which the density of excited ions along the amplifier is determined. With this value, we can obtain the gain coefficient and noise power spectral densities along the amplifier.

In the second method, every channel inside the EDFA corresponds to a different signal. Both directional types of ASE are divided into bandwidth sections, each associated with a signal. The differential system is composed of the propagation equations for each signal. From the solution of this system, the gain coefficient and noise power spectral densities along the amplifier are obtained.

In the first method, the computational time increases with the number of sections. The number of points necessary to simulate noise and propagation effects in the pulses is always higher than the amount of points used in the photon flux conservation method. On the other hand, the Runge–Kutta algorithm used in the second method can be easily adapted to match the number of points needed by the SSFM without significantly increasing the calculation time.

All numerical simulations presented in this article refer to a Type III (germano-aluminosilicate) EDFA whose emission ($\sigma_e$) and absorption ($\sigma_a$) cross sections are shown in Fig. 1 [2]. With these cross sections, it is possible to define the cross-section ratio as well as the gain and loss spectra, respectively, as

$$
\eta(\omega) = \frac{\sigma_e(\omega)}{\sigma_a(\omega)} \\
\gamma(\omega) = \frac{\sigma_e(\omega)}{\sigma_a(\omega)} \Gamma(\omega) \rho \\
\alpha(\omega) = \frac{\sigma_a(\omega)}{\Gamma(\omega) \rho}
$$

where $\rho$ is the ion density and $\Gamma(\omega)$ is the overlap integral between the optical mode and the spatial distribution of erbium ions.

Considering that the erbium ions are confined to the fiber core with a homogenous distribution, and assuming a Gaussian approximation for the optical field distribution, then [2] $\Gamma(\omega)$ is

$$
\Gamma(\omega) \approx 1 - \exp\left(-\frac{q_0^2}{\gamma_0^2(\omega)}\right)
$$

where $q_0$ is the fiber core radius, and $\gamma_0$ is the spot-size radius of the optical mode [4].

In our model of the amplifier, there are two energy levels with erbium densities $N_1$ and $N_2$, respectively, with $N_1 + N_2 = \rho$.

The gain coefficient $g(z, \omega)$ is given by

$$
g(z, \omega) = \left[\alpha(\omega) + \gamma(\omega)\right] \frac{N_2(z)}{\rho} - \alpha(\omega).
$$
The populations of erbium ions in each level ($N_1$ and $N_2$) can only be determined after the power evolutions of all channels are known. The normalized power of any signal is

$$p_k = \frac{\alpha_k + \gamma_k}{s} \frac{P_k}{\eta \omega_k}$$

(6)

where

- $\alpha_k$, $\gamma_k$: cross sections at the signal frequency $\omega_k$;
- $s$: saturation parameter;
- $\eta$: reduced Planck constant;
- $P_k$: actual signal power.

The cross-section ratio at the signal frequency is $\eta_k = \gamma_k / \alpha_k$.

Every data and pump channel is represented as a different signal. There will also be several other channels associated with noise in the transmission bandwidth.

To describe the noise in the transmission bandwidth, we separate the forward- from the backward-propagating noise. We also divide the ASE bandwidth in $N_{AS}^*$ segments, each one with a frequency width $\Delta f_{eff}$. Each segment corresponds to a noise channel (i.e., $p_k^{ASE^+}$ and $p_k^{ASE^-}$ are the $k$th components of the ASE noise spectrum in the forward and backward directions, respectively). To obtain the noise power spectral density $S_n(z, \omega)$, we divide each noise channel by its effective width

$$S_n^\pm(z, \omega) = \frac{N_{AS}^*}{N} p_k^{ASE^\pm} \frac{1}{2\pi \Delta f_{eff}} \left(1 + \eta_k^{ASE^\pm}\right) \left(\omega - \omega_k^{ASE^\pm}\right)$$

(7)

Since the noise power spectral density is a continuous function and we have a limited number of channels, we assume that $S_n$ is constant inside each channel. Hence, a compromise has to be reached between the number of noise channels and their effective width.

The evolution of signal power along the amplifier is described by the following system of differential equations [2]:

$$\frac{\partial p_k}{\partial z} = \frac{\pm \alpha_k \eta_k}{1 + \sum_{k=1}^{N} \eta_k + \sum_{l=1}^{N_{AS}^*} \left(p_l^{ASE^+} + p_l^{ASE^-}\right)} \times \left[-1 + \sum_{l=1}^{N} \frac{\eta_k - \eta_l}{1 + \eta_l} p_l^{ASE^+} + \frac{\eta_k - \eta_l}{1 + \eta_l} p_l^{ASE^-}\right]$$

(8)

$$\frac{\partial p_k^{ASE^\pm}}{\partial z} = \frac{-\alpha_k^{ASE^\pm}}{1 + \sum_{k=1}^{N} \eta_k^{ASE^\pm} + \sum_{l=1}^{N_{AS}^*} \left(p_l^{ASE^+} + p_l^{ASE^-}\right)} \times \left[\left(\sum_{l=1}^{N} \eta_k^{ASE^\pm} + \eta_l^{ASE^\pm}\right) - \eta_k^{ASE^\pm} \frac{2\Delta f_{eff}}{s}\right] \frac{\eta_k - \eta_l}{1 + \eta_l} p_l^{ASE^+} + \frac{\eta_k - \eta_l}{1 + \eta_l} p_l^{ASE^-}\right]$$

(9)

where $N$ is the total number of data and pump channels. In (8) and (9), the $\pm$ sign should be chosen whether the signal is propagating along the positive or negative longitudinal directions, respectively.

After calculating the power for all the signals, we can evaluate the population of erbium ions in energy level 2

$$N_2(z) = \rho \left[\sum_{k=1}^{N} p_k^{ASE^+} + \sum_{l=1}^{N_{AS}^*} p_l^{ASE^+}\right]$$

(10)

The gain profile is then obtained through (5).

III. OPTIMIZATION PROCEDURE

In this section, we present an optimization procedure to determine some relevant parameters of the fiber amplifier. The system analyzed has a forward pumping scheme in order to speed up the numerical convergence of the algorithm. One should stress, however, that this model can be easily applied to systems with other pumping schemes.

We present, in Table I, the amplifier parameters that we will use henceforth. The pump power and amplifier length must be determined by an optimization procedure. This optimization consists in finding, for a given pump power, the length of the amplifier where the gain is maximized. This is equivalent to finding the position where the normalized power of the data channel $p_d$ is such that

$$\frac{\partial p_d}{\partial z} \bigg|_{z=L_{opt}} = 0.$$  

(11)

If we consider only one signal, one pump, and disregard noise, an expression for $L_{opt}$ is available [2]. However, when the noise is included, it is not possible to find a closed-form expression for the optimal length.

In Fig. 2, we show the maximum gain for the data channel obtained for different values of pump power, with and without noise. The gain does not have a linear dependence with the pump power. There is a value for the pump power above which the amplifier begins to saturate. Any increase in the pump power will only bring a marginal increase in the gain. From Fig. 2, one may see that, for low values of pump power ($P_{pump} < 5$ mW), the presence of noise does not affect the final gain. For
higher pump powers, the increasing ASE noise along the amplifier causes self-saturation, thus reducing the gain.

The optimum amplifier length is shown in Fig. 3, as a function of the pump power. The optimum length of a noise-free amplifier is greater than for a noisy one. This happens since the pump can maintain the amplification demand for greater distances in the absence of noise.

In Fig. 4, we show the pump, data, and ASE channels along the 9-m long amplifier for an initial pump power of 3 mW. The two ASE channels are the forward and backward noise at the wavelength of the data channel.

IV. Numerical Algorithm

In this section, we present our numerical algorithm, which is a modified version of the SSFM [7]. In fact, the model that we use for pulse propagation inside the EDFA is based on the nonlinear Schrödinger equation that includes the gain and noise of the amplifier.

The SSFM simulates the propagation in small iterative steps of size \( h \), applying at each consecutive step the nonlinear and the dispersive effects separately. The slowly varying envelope amplitude of the electric field at \( \zeta + h \), \( u(\zeta + h, T) \), is obtained from \( u(\zeta, T) \). When the transmission over an active medium is involved, one has to include two new terms: the noise generated in each amplifier step \( \hat{n}(\zeta, \Omega) \) and the corresponding gain coefficient \( g(\zeta, \Omega) \). The iterative procedure can be summarized as follows:

\[
\begin{align*}
  u'_m &= u(0) \exp \left( iN_{\text{NL}} z |u(m)|^2 h \right) \\
  U(m) &= \int_{-\infty}^{\infty} u'_m e^{i\xi^2 T} d\xi \\
  U'_m &= U(m) \exp \left\{ \frac{h}{2} \left[ g(\zeta, \Omega)L_D z - \xi^2 \gamma^2 \right] \right\} + \hat{n}(\zeta, \Omega) \\
  u(m+1) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} U'_m e^{-i\xi T} d\xi
\end{align*}
\]

where \( u \) is the pulse envelope. Initially, in each step \( u(0) = u(\zeta, \tau) \) and, after obtaining convergence, the pulse envelope at \( \zeta + h \) is

\[
u(\zeta + h, \tau) = u(m+1) \exp \left( -\frac{\alpha_L}{2} \frac{L_D z}{h} \right)
\]
where $\alpha_T$ is the attenuation coefficient. We have disregarded the attenuation coefficient when propagating inside the amplifier.

Since the presence of noise modifies the nature of the problem to a stochastic one, an iterative method has to be used in each step in order to assure a more stable convergence than the usual SSFM [13]. Although higher order methods can be used, we have found that the presented algorithm achieves sufficient accurate results. This iterative method is only used in the amplifier.

Equations (12)–(16) were written in the usual soliton units [7]

\begin{align*}
\zeta &= \frac{\Delta z}{L_{Dx}} \\
T &= t - \beta_1 x \\
\Omega &= \omega - \omega_0
\end{align*}

where $L_{Dx} = \frac{\tau_0^2}{|\beta_2|}$ is the dispersion length of medium $x$ (i.e., $L_{Dx}$ and $L_{Df}$ are the dispersion lengths of the fiber amplifier and of the undoped fiber, respectively). The soliton full-width at half-maximum (FWHM) is $\tau_s = 1.763\tau_0$, and $\beta_1$ and $\beta_2$ are the first and second derivatives of the longitudinal wavenumber, calculated at the carrier frequency $\omega_0$. The nonlinear coefficient $N_{NL,x}$ also varies according to the dispersion length of the medium [4]. In (14), one should disregard gain and noise (i.e., $g(\zeta, \Omega) = \hat{n}(\zeta, \Omega) = 0$) whenever the pulse is propagating in a conventional fiber.

To generate the noise vectors $\hat{n}$ we assume that there is only forward-propagating noise. The reason for this simplification is the impossibility of adding a backward signal to the SSFM, which is a forward-propagating algorithm.

### A. Noise Description

In each iterative step, the total output noise spectrum $\hat{n}_T(\zeta, \Omega)$ is the sum of the amplified input noise plus the internally generated term $\hat{n}(\zeta, \Omega)$. Hence

$$\hat{n}_T(\zeta + h, \Omega) = \hat{n}(\zeta, \Omega) + \hat{n}_T(\zeta, \Omega) \exp\left(\frac{1}{2}g(\zeta, \Omega)L_{Dx}h\right).$$

We assume that, at the beginning of the amplifier, $\hat{n}_T(\zeta, \Omega) = 0$. The noise spectrum must always obey the following statistical relations [12]:

\begin{align*}
\langle \hat{n}_T(\zeta, \Omega) \rangle &= 0 \\
\langle \hat{n}_T(\zeta, \Omega) \hat{n}_T(\zeta, \Omega) \rangle &= 0 \\
\langle \hat{n}_T(\zeta, \Omega) \hat{n}_T(\zeta, \Omega) \rangle &\approx 4\pi^2 S_\Omega(\zeta, \Omega)\delta(\Omega - \Omega_1)
\end{align*}

where $\langle \rangle$ ensemble average; $\delta(\Omega)$ Dirac’s delta function; $S_\Omega(\zeta, \Omega)$ noise power spectral density according to (7).

To observe (21) and (22), the noise spectrum generated in each iterative step (i.e., $\hat{n}(\zeta, \Omega)$) should have independent frequency components, each one with a uniform phase distribution and a Poisson amplitude distribution. Each Poisson parameter is calculated so that the total noise spectrum—determined using (20)—satisfies (23). The use of the Poisson distribution is based on the assumption that the noise generated in each fiber step obeys a Markov immigration process [17].

### B. System Description

In order to show the use of the presented model we have simulated a communication system based on the average-soliton regime. Therefore, the distance between amplifiers must be smaller than the dispersion length [4]. In the average-soliton regime, the pulse’s peak power results from a balance between the nonlinear and the dispersive effects. The parameters used for the solitons and the system are presented in Table II.

At the beginning of the fiber, the soliton has the form

$$u(\zeta = 0, T) = A_0 \text{sech}\left(\frac{T}{\tau_0}\right)$$

where $A_0 = 3.82$ dB is the pre-emphasis value [4].

The soliton peak power value presented in Table II was determined in order to assure the amplifier input data power $P_d$

$$P_{\text{peak}} = \frac{P_d^2 I_{\text{pump}} \exp\left(\frac{\alpha_T L_a}{\tau_0}\right)}{\tau_0} = 0.22 \text{ mW}.$$  

The minimum gain needed to compensate the attenuation in each fiber span is 9.2 dB. If one adds the typical losses from the amplifier’s components [16] (e.g., isolators, filters, multiplexers), the necessary gain increases to 12.1 dB. The amplifier that presents this gain has a pump power of $P_p = 3$ mW (see Fig. 2) and a length of $L = 9$ m (see Fig. 3).

In Fig. 5, we show the evolution of an initially noise-free soliton within the amplifier. In Fig. 6, we show the pulse propagation along a chain of ten amplifiers. Both figures are normalized to the maximum amplitude of the pulse. Fig. 5 corresponds to each amplification stage in Fig. 6. As the average-soliton regime is observed, the form of the pulse remains unchanged after each amplification step (the small perturbations are noise related).

### V. RANDOM WALK AND COMPARISON WITH THE GORDON–HAUS EFFECT

In this section, we use the developed numerical algorithm to show some relevant features of soliton propagation inside EDFAs—namely, the random walk due to ASE that causes the Gordon–Haus effect.

According to Fig. 6, noise accumulates after each iterative step, thus reducing the signal-to-noise ratio. Moreover, noise
also induces a random deviation of the central frequency of the pulses. As any fiber is a dispersive medium, each soliton will suffer a fluctuation in its group velocity and, hence, there is an uncertainty in its arrival time at the detector. This problem is the so-called Gordon–Haus effect [11], [12].

We have studied the frequency jitter that appears during soliton propagation within the amplifier. At the entrance, the pulse is considered noise-free and, hence, it has the usual soliton shape. The pulse energy and the average frequency deviation are

$$E(\zeta) = \int_{-\Omega_{hm}}^{\Omega_{hm}} P_{\text{peak}} U^2(\zeta, \Omega) d\Omega$$  

$$\Omega(\zeta) = \frac{1}{E(\zeta)} \int_{-\Omega_{hm}}^{\Omega_{hm}} \Omega P_{\text{peak}} |U(\zeta, \Omega)|^2 d\Omega$$  

where $U'$ is the normalized pulse (in the spectral domain) propagating in a noise-free amplifier and $U$ is the corresponding pulse with noise according to (14). The limits of the integrals were chosen so that the pulse energy at the beginning of the amplifier would be more than 99% of the total value ($\Omega_{hm} = 2.4/\tau_0$). The average square of $\Omega$, when the pulse is propagating in the amplifier, is estimated, according to the Gordon–Haus effect, by [12]

$$\langle \Omega^2(\zeta) \rangle = 4\pi^2 \frac{2S_n(\zeta)}{3\pi^2 E(\zeta)}.$$  

The definition of the Fourier transformation (different from the one used in [12]) is reflected by the term $4\pi^2$ in (28). In order to obtain this equation, one has to assume that the gain and the noise power spectral density are constant in the frequency domain, and that the dispersion and the nonlinear effects within the amplifier are negligible. Due to the model used, we must modify the noise power spectral density to a frequency average in order to use it in (28)

$$S_n(\zeta, \Omega) = \frac{1}{2\Omega_{hm}} \int_{-\Omega_{hm}}^{\Omega_{hm}} S_n(\zeta, \Omega) d\Omega.$$  

In Fig. 7, we compare (28) with the average square of $\Omega$ from (27), simulated with the modified SSFM after 10,000 different runs. The vertical axis is the relation between the square root of $\langle \Omega^2 \rangle$ and the spectral width at half maximum for an ideal soliton, where $\Omega_0 = 3.5\Omega_{hm}/(\tau_0\pi)$. Due to the good agreement between both curves in Fig. 7, we conclude that the dispersive and nonlinear effects in the amplifier have no relevant effect on frequency jitter in the present case. This can be explained by the small length of the amplifier compared to the characteristic distance of those two effects, $L_{\text{cha}} = 20$ and $L_{\text{cha}} = 200$ km.

In [12], the average of $\Omega$ was zero, and hence (28) was also the variance of the frequency shift. However, in this model, neither the gain nor the noise power spectral density are constant in frequency and so the central frequency will shift to the side of the spectrum where the gain is higher.

As shown in Fig. 8, the average of $\Omega$ is not zero, but those values are ten times smaller than those in Fig. 7. Therefore, in this system, the average of $\Omega$ is almost the same as its variance. One should note, however, that when: 1) the gain has a steeper slope at the carrier frequency; 2) the pulses are shorter; or 3) the signal-to-noise ratio is higher, then we must compare (28) with the variance of $\Omega$.

We have also analyzed the timing jitter that was generated, during the process of soliton amplification, within the fiber amplifier. Outside the EDFA, the timing jitter is mainly due to the group-velocity shift associated with frequency jitter, as is well
known from the Gordon–Haus effect. However, for the special case herein presented, the timing jitter generated inside the amplifier was mainly due to noise itself rather than to the frequency jitter through the effect of group-velocity dispersion. The reason for this result is that, for the case under study, the amplifier length is negligible in comparison to the dispersion length.

VI. WDM SOLITON SYSTEM

In this section, we present the results obtained with a four-channel WDM soliton system using the characteristics described in Table II. The wavelengths of the four channels are: \( \lambda_1 = 1551 \text{ nm} \), \( \lambda_2 = 1551.5 \text{ nm} \), \( \lambda_3 = 1552 \text{ nm} \) and \( \lambda_4 = 1552.5 \text{ nm} \). Each channel has a bit rate of 10 Gbit/s and the input pulse is

\[
\psi(\zeta = 0, T) = \sum_{k=1}^{4} A_k \text{sech} \left( \frac{T}{\tau_0} \right) \exp \left[ -i2\pi c \left( \frac{1}{\lambda_k} - \frac{1}{\lambda_0} \right) T \right] \tag{30}
\]

where \( \lambda_0 = 1551.75 \text{ nm} \) is the normalization wavelength.

The amplifier length and the pump power were changed in order to ensure the necessary 12.1 dB gain to compensate for the losses. In fact, this gain is wavelength dependent and, therefore, is only assured for channel 1. For all the other channels, the gain is slightly higher. For this system, the amplifier is \( L = 9.5 \text{ m} \) long and the pump power is \( P_p = 3.72 \text{ mW} \).

In Fig. 9, we show the normalized spectral evolution of the four channels along a chain of ten amplifiers. The noise increase is clearly visible after the first amplifier. Also visible is the different gain of the four channels. As expected from the amplifier’s cross sections, channel 4 is the one with higher gain, while channel 1 has the lower gain.

In longer chains, the fourth channel would cause the amplifier to saturate, therefore limiting the gain of the remaining channels. To solve this problem, several techniques have been used to flatten the gain profile of the amplifier [18], [19].

VII. CONCLUSION

We have presented a numerical algorithm, based on a modified version of the SSFM, to analyze soliton propagation inside an EDFA. The actual gain profile is determined by considering the measured cross sections of the amplifier. We have also shown that this algorithm is well suited for WDM soliton communication systems. Both noise in EDFAs and soliton propagation in noise-free optical amplifiers have been considered in the literature. However, the effect of ASE on soliton propagation in the amplifier was analyzed in this article—as far as the authors are aware—for the first time, although only forward ASE was considered.

The amplifier length was determined through an optimization procedure that includes the effect of ASE. The results using this procedure were not too different from the noise-free case: a difference of 0.5 dB in the maximum gain and of 0.13 m in the amplifiers length was registered for \( P_p = 3 \text{ mW} \). However, for amplifiers with higher gain, noise should not be neglected in the optimization procedure.

A frequency jitter was generated during the amplification process as expected. After comparing our results with those presented in [12], we conclude that the amplifier length was too small for the dispersive or nonlinear effects to have any significant influence within the active device. Nevertheless, in larger fiber amplifiers—such as in distributed amplification—the discrepancy between the present algorithm and the Gordon–Haus effect is expected to be more significant.

There is an average shift in the central frequency of the soliton-like pulses caused by nonflat spectral gain and noise power spectral density profiles. In the numerical case presented here, the timing jitter generated within the amplifier was mainly produced by noise itself than by the group-velocity shift associated with frequency jitter. However, outside the amplifiers and for reasonable amplifier spacing, the timing jitter is mostly determined by frequency jitter as stated in the Gordon–Haus limit.
One should finally stress that the method herein presented disregards backward-propagating ASE, since it is based on the SSFM that is a forward-propagating scheme. Hence, in such applications where backward-propagating noise is relevant, an alternative algorithm should be developed.

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Carlos R. Paiva (S’84–M’90) was born in Lisbon, Portugal, in 1957. He received the Licenciado degree in electrical engineering from Instituto Superior Técnico (IST)—Universidade Técnica de Lisboa, Lisbon, Portugal, in 1974. He is currently a Research Assistant at the Instituto de Telecomunicações, Lisbon, Portugal. His present research interests include electromagnetics of complex media (namely chiral and omega-media) and its applications to soliton communication systems and all-optical switching, fiber amplifiers and semiconductor lasers.

Jorge R. Costa (M’99) was born in Lisbon, Portugal, in 1974. He received the Licenciado degree in electrical engineering from Instituto Superior Técnico (IST)—Universidade Técnica de Lisboa, Lisbon, Portugal, in 1997. He is currently working toward the Ph.D. degree at IST, where his dissertation work concerns soliton amplification in erbium-doped fibers.

He is currently a Research Assistant at the Instituto de Telecomunicações, Lisbon, Portugal. He is also a lecturer at the Department of Work and Information Sciences, Instituto Superior das Ciências do Trabalho e da Empresa (ISCTE), Lisbon, Portugal.

Afonso M. Barbosa (S’80–A’83–SM’90) was born in Coimbra, Portugal, in 1950. He received the Licenciado degree in electrical engineering from Instituto Superior Técnico (IST)—Universidade Técnica de Lisboa, Lisbon, Portugal, in 1972, the Master’s degree in electronic engineering from NUFFIC, The Netherlands, in 1974, and the Ph.D. degree in electrical engineering from IST in 1984.

He is currently a Full Professor and Head of the Department of Electrical and Computer Engineering of IST, and a member of the Research Staff and Executive Board of Instituto de Telecomunicações, Lisbon, Portugal. His current research interests are in electromagnetic wave theory and applications, namely microwave, millimeter-wave, and optical waveguide structures, antennas, fiber amplifiers, and scattering with emphasis on complex media.
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